**Q1** [14pts]

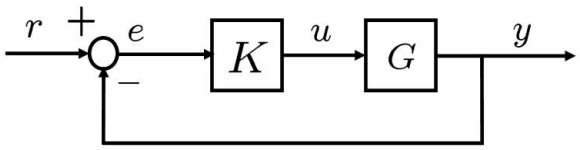



Figure 1: A feedback interconnection

Consider the feedback interconnection shown in Figure 1 where $G$ and $K$ are proper transfer functions. Here it is assumed that a transfer function is of the form $\frac{n(s)}{d(s)}$ where $n$ and $d$ are polynomials in $s$ with no common factors. For the following questions answer if the statement is true or false. If true provide an example and if false provide a proof.

1. [7pts] There exist single-input, single-output proper transfer functions $G$ and $K$ such that $\frac{1}{1+GK}$ and $\frac{G}{1+GK}$ are stable but $\frac{K}{1+GK}$ is not.
2. [7pts] There exist single-input, single-output proper transfer functions $G$ and $K$ such that $\frac{K}{1+GK}$ and $\frac{G}{1+GK}$ are stable but $\frac{1}{1+GK}$ is not.

**Q2** [15pts]

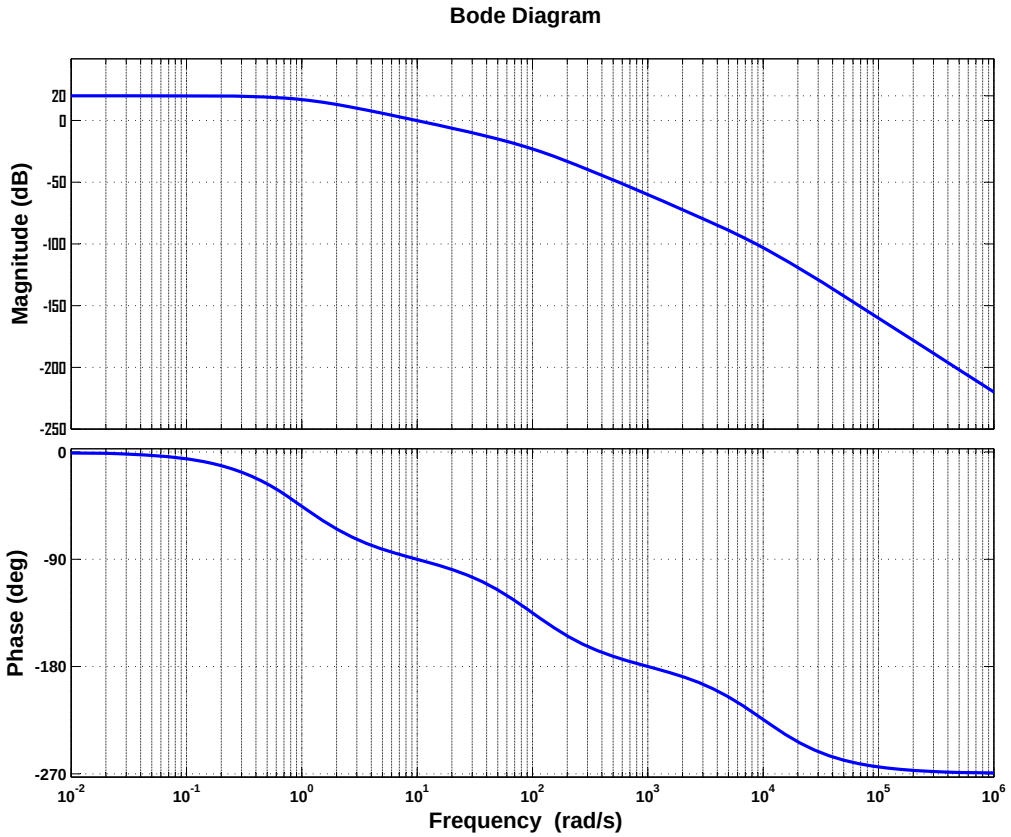



Figure 2: Bode-plot for Problem 2

Consider the bode plot of a minimum-phase transfer function $G(s)$ (the bode plot shows in the magnitude plot $20\log_{10}|G(j\omega)|$ in db on the Y axis).

1. [3pts] Draw the asymptotes on the bode plot. Use the asymptotes to determine the transfer function $G(s)$.

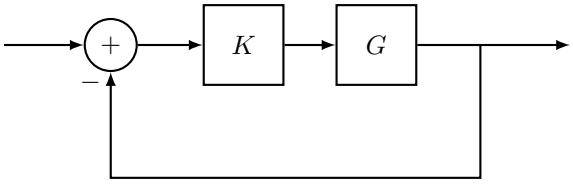
2. [3pts] (a)Determine the gain-crossover frequency ($\omega_{gc}$) and the phase-crossover frequency ($\omega_{180}$). (b) Determine the phase and gain margin.

3. [3pts] Suppose the plant $G$ is in a unity negative feedback interconnection with a controller $K$ (see Figure 1). With the controller $K = k_p$ a positive real constant, find the smallest value of $k_p$ such that the interconnection shown is unstable. (Hint: Use the gain margin to obtain the result).

4. [3pts] With $K = 1$ determine the steady state error due to a step input for the interconnection shown. Also, determine the steady state error due to a ramp input.

5. [3pts] Design a Proportional Integral (PI ) controller, $K = k_p + \frac{k_I}{s}$, to increase the type with specifications (i) the gain crossover frequency has to be 100 rad/sec (ii) the phase margin has to be at least 40 degrees.

**Q3** [11pts]




Consider the unity gain loop depicted above, with open loop transfer function given by $KG(s) = K\frac{s+1}{s(s-1)}$. Let $K = k$ be a constant gain. Find the range of $k$ that give phase margins of at least $30°$.